

**MECHANICS 3 (A) TEST PAPER 3 : ANSWERS AND MARK SCHE**

1. (a) Total energy is conserved throughout, so  $v^2 = gr$  M1 A1  
 (b)  $T = \frac{mv^2}{r}$   $\frac{1}{2} mu^2 = m(2gr - gr) + \frac{1}{2} mgr$   $u^2 = 3gr$   $T = 3mg$  M1 A1 M1 A1 A1 7
2. (a)  $T = \frac{2\pi}{n}$   $\frac{\pi}{2} = \frac{2\pi}{n}$   $n = 4$  M1 A1  
 $v^2 = n^2(a^2 - x^2)$   $9 = 16(a^2 - 1)$   $a = 1.25$  m M1 A1  
 (b)  $a_{max} = an^2 = 1.25 \times 16 = 20$  ms<sup>-2</sup> B1  
 (c)  $x = a \sin nt$   $0.25 = 1.25 \sin 4t$   $\sin 4t = 0.2$  M1 A1  
 Least  $t$  when  $4t = \arcsin(0.2)$   $t = 0.05$  s M1 A1 9
3. (a) Grav. P.E. loss = E.P.E. gain :  $3mgl = \frac{\lambda}{16l} (10l - 8l)^2$   $\lambda = 12 mg$  M1 A1 M1 A1  
 (b) P.E. loss = (K.E. + E.P.E.) gain :  $2mgl = \frac{1}{2} mv^2 + \frac{12mg}{16l} (4\sqrt{5}l - 8l)^2$  M1 A1  
 $4gl = v^2 + 24gl(1 - 4\sqrt{5})$   $v^2 = 4(24\sqrt{5} - 53)gl$  M1 A1 A1 9
4. (a)  $F = ma$  :  $0.8v \frac{dv}{dx} = -\frac{0.8rv^2}{1+x^2}$   $\frac{dv}{dx} = -\frac{rv}{1+x^2}$  M1 A1  
 (b)  $\int \frac{dv}{v} = -\int \frac{r dx}{1+x^2}$   $\ln v = -\frac{1}{2} \ln(1+x^2) + c$   $c = \ln 2$  M1 A1 M1 A1 A1  
 When  $x = 1$ ,  $\ln v = \ln 2 - \frac{1}{2} \ln 2 = \frac{1}{2} \ln 2$   $v = \sqrt{2}$  M1 A1 9
5. (a) Centre of mass of cone is  $\frac{1}{4}h$  from base along axis B1  
 Let  $POQ = 2\alpha$ .  $M(O) : T(2r \cos \alpha) = \frac{3}{4}h \sin \alpha$  B1 M1 A1 A1  
 $T = \frac{3h}{8r} \tan \alpha$  But  $\tan \alpha = \frac{r}{h}$  so  $T = \frac{3}{8} mg$  B1 M1 A1  
 (b) Vertically :  $S + T = mg$   $S = \frac{5}{8} mg$  M1 A1 10
6. (a) For  $Q : T = mg$  For  $P : T = m(0.2)\omega^2$  B1 M1 A1  
 $\omega^2 = g/0.2 = 49$   $\omega = 7$  No. of r.p.m. =  $\frac{7}{2\pi} \times 60 = 66.8$  A1 A1  
 (b) For  $Q : T \sin 45^\circ = m(0.2)\omega_1^2$ ,  $T \cos 45^\circ = mg$  M1 A1 A1  
 $\tan 45^\circ = 0.2\omega_1^2 / g$   $\omega_1^2 = 49 \tan 45^\circ = 49$  M1 A1  
 For  $P : T = m(0.2)\omega^2$  But  $T = mg\sqrt{2}$  so  $\omega^2 = 49\sqrt{2}$  M1 A1  
 $\omega^2 : \omega_1^2 = \sqrt{2} : 1$   $\omega : \omega_1 = 2^{1/4} : 1$  M1 A1 14
7. (a) At distance  $x$  from  $O$ ,  $T = -mx$   $T = \frac{kmg}{l}$  B1 B1  
 Hence  $\ddot{x} = -\frac{kg}{l}x$ , so motion is S.H.M. M1 A1  
 (b) Loss of E.P.E. = gain in K.E. :  $\frac{kmg}{2l} \left(\frac{9}{16}l^2\right) = \frac{1}{2} mgl$   $k = \frac{16}{9}$  M1 A1 A1  
 (c) K.E. dissipated = work done against friction :  $\frac{1}{2} mgl = \mu mgx$  M1 A1  
 $x = \frac{l}{2\mu}$  which must be  $< l$ , so  $\mu > \frac{1}{2}$  M1 A1  
 (d) Time from  $O$  to stop is  $t$  where  $0 = \sqrt{gl} - \frac{3}{4}gt$   $t = \frac{4}{3}\sqrt{\frac{l}{g}}$  M1 A1  
 $\frac{1}{4}$  period =  $\frac{1}{4} \left( \frac{2\pi}{\sqrt{16g/9l}} \right) = \frac{3\pi}{8} \sqrt{\frac{l}{g}}$  Total time =  $\left( \frac{3\pi}{8} + \frac{4}{3} \right) \sqrt{\frac{l}{g}}$  M1 M1 A1 A1 17